Investigating Reversing Motion of Truck-Semitrailer Along Clothoid Curve

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Abstract. The focus of the research is on the most common transport vehicle, the truck-semitrailer combination. A single-track kinematic model is used, supplemented by the dynamics of the steering system, to design a linear state feedback controller for the low-speed path-following problem in reverse. Despite the significant time delay considered in our study, the linear stability of the system is ensured, even for complicated maneuvers. The advantages of the adaptive gain tuning method are presented based on simulations of the commonly used 90-degree alley dock.

Keywords: truck-semitrailer, reverse maneuvering, time delay, path-following

1 Introduction

Autonomous vehicles imply a fundamental change in modern engineering. Optimizing safety, speed, and operating costs is the task and responsibility of engineers. Beyond safety, the competition among automotive designers now focuses on reducing travel time and costs, even more so, in the case of commercial trucks. Key tasks on highways involve operating long convoys with only one human driver in the front, i.e., platooning. Vehicle-to-vehicle communication decreases fuel consumption through real-time traffic data.

Maneuvering articulated vehicles, especially in reverse motion, demands special attention due to potential accidents like the Jackknifing phenomenon [1, 2]. Driver Assistance Systems (DAS) are available, but these features only assist human drivers, not replace them in complicated situations. Auto-parking systems at docking stations address the dwelling time problem of transport companies [3]. The implementation of automated maneuvers cannot be realized without developing path-following controls in reverse motion [4]. Installing fully automated parking systems into loading bays allows drivers to spend more time on the road.

Our research focuses on the most common freight transport vehicle, the truck-semitrailer combination. Due to the dwelling time problem, we are interested in realizing complicated maneuvers in reverse motion. The single-track kinematic model is applied with the assumption of rigid wheels. The paper presents and analyzes a control scheme for solving the path-following problem in low-speed reverse motion. Neglecting the time delay of localization and control-related computations on stability may be irresponsible; thus, the stability of reversing along curves with constant curvatures is investigated in the presence of significant time delay. Based on stability charts – produced by the semi-discretization method, see [5] – related to reversing along circular arcs (constant curvature), three different gain tuning strategies for varying curvature are selected, and their performances are compared via simulations.

2 Model setup

The vehicle combination is modeled with the single-track kinematic model, see Fig. 1. The wheels at points F, R and T are assumed rigid. The constant longitudinal speed of the rear axle of the towing truck is V. The geometry is described by the wheelbase l of the towing vehicle, the distance a between the kingpin (K) and the rear axle (R), and the length L of the trailer's axle measured from the kingpin. During the path-following problem, the prescribed point of the vehicle combination is the center point T of the trailer's axle. Table 1 contains the numerical values of the parameters.

The vector of the state variables is $\mathbf{q} = \left[e\ \Theta\ \varphi\ \delta\ \omega\right]$, where e is the lateral deviation between the prescribed point of the vehicle combination (T) and the closest point on the desired path (C); Θ denotes the yaw angle error; and φ is the relative yaw angle between the two vehicle parts. The dynamics of the steering system are considered using the steering angle δ of the front wheel (which is related the only actuation) and steering rage ω. The control gains p and d of the steering servo control are listed in Table 1.

The equations of motion are derived from the kinematic constraints of the vehicle. On the one hand, the velocity vectors of points F, R and T are parallel to the related wheel planes, described by the unit vectors $\mathbf{e}_{\mathrm{F}}$, $\mathbf{e}_{\mathrm{R}}$ and $\mathbf{e}_{\mathrm{T}}$ in Fig. 1. On the other hand, the longitudinal speed of the truck's rear axle (point R) is constant (V). Finally, the equations of motion are formulated in

$$\dot{e} = V\left(\sin(\theta-\varphi) - \frac{a}{l}\tan\delta\cos(\theta-\varphi) + \cos\theta\left(\sin\varphi + \frac{a}{l}\cos\varphi\tan\delta\right)\right), \quad (1)$$

$$\dot{\theta} = \frac{V}{l}\tan\delta + \dot{\varphi} - \kappa\dot{s}, \quad (2)$$

$$\dot{\varphi} = -\frac{V}{lL}\left(l\sin\varphi + (L + a\cos\varphi)\tan\delta\right), \quad (3)$$

$$\dot{\delta} = \omega, \quad (4)$$

$$\dot{\omega} = -p\delta - d\omega + p\delta_{\mathrm{des}}, \quad (5)$$

The reverse path-following motion is achieved by a linear feedback controller:

$$\delta_{\mathrm{des}}(t) = \delta_{\mathrm{ff}}(\kappa) - P_e\, e(t-\tau) - P_\theta\, \theta(t-\tau) - P_\varphi\left(\varphi(t-\tau) - \varphi^\star\right), \quad (6)$$

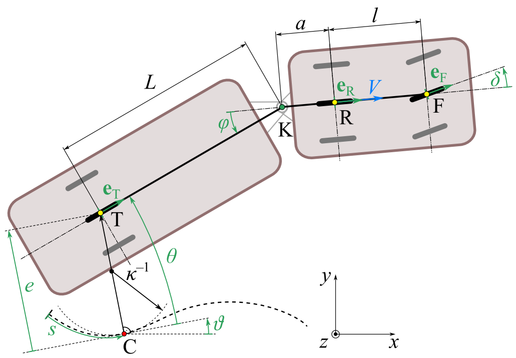

Fig. 1: Single-track kinematic model of the truck–semitrailer.

Table 1: Numerical values of the parameters.

Parameter	Value
V	-1.5 m/s
a	-0.8 m
l	3.5 m
L	10 m
p	300 1/s^2
d	34.6 1/s

where τ is the feedback delay; P_e, P_θ and P_φ are the control gains (representing the feedback term) related to the lateral deviation, the angle error and the relative yaw angle between the truck and the trailer, respectively. The feedforward term $\delta_{\mathrm{ff}}(\kappa)$ and the reference value $\varphi^\star(\kappa)$ of the relative yaw angle are the steady-state solutions of (1)–(5), which depend on the curvature κ, of course. Note that the steady-state solution could be generated from geometric considerations as well.

3 Stability and optimization

Fig. 2(a) shows the linear stability chart of the path-following motion, when the time delay is $\tau = 0.5$ s. Colored areas represent the stable gain configurations; white crosses denote the most stable gain setup (in the sense of the smallest real part of the rightmost eigenvalues) for each curvature value. As can be seen, the most stable gain setup for one specific case may be located outside of the stable domain for a different curvature value. However, reversing along a path with varying curvature may still be possible if control gains are selected from the intersection of stable domains (assuming that the rate of change is relatively small).

The main contribution of this study is the comparison of control gain tuning methods. One of the most common maneuvers in loading bays is the so-called 90-degree alley dock in reverse. This maneuver can be realized along a path consisting of three clothoid arcs (i.e., the curvature κ changes linearly, see [6]), and a straight-line run-out at the end. The desired position and orientation of the trailer, and the curvature of the path can be adjusted at the start and end points of the maneuver. The trajectory with the explanation of the segments is plotted in Fig. 2(b).

The differences in the performance between three different gain tuning methods are shown in Fig. 3. The comparison is based on simulation results by solving the nonlinear equations of motion (1)–(4) using the *Matlab* built-in solver *dde23*

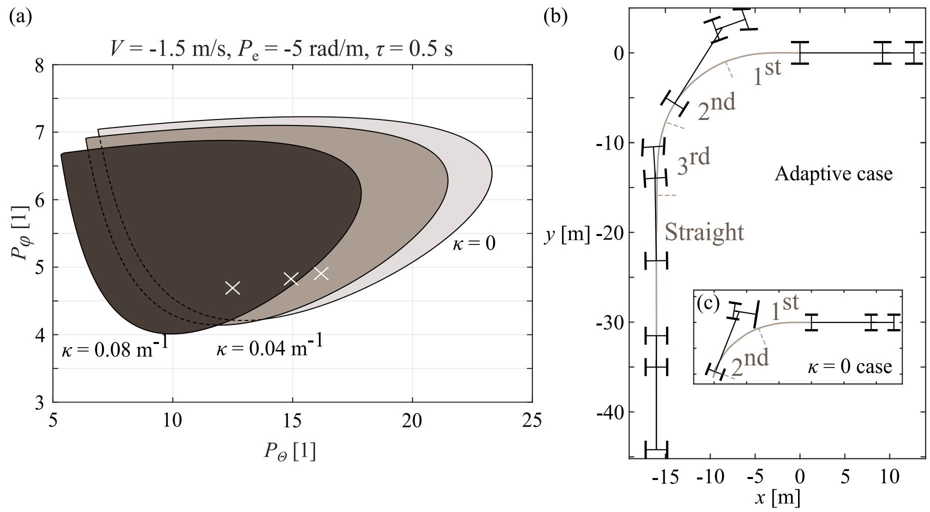

Fig. 2: (a) Stability chart in the plane of control gains. Shaded areas denote the stable domains for different curvature values. White crosses represent the most stable gain setups within each stable region. (b)–(c) Trajectories of the trailer's axle (prescribed point) during the investigated 90-degree alley dock with the skeleton model of the vehicle combination at some specific time steps. The clothoid segments and the straight run-out are labeled along the trajectories. [LEHETNE EGY NYILAT TENNI A TOLATASI IRANYT MUTATVA]

with adaptive time stepping. The steady-state solution related to zero curvature (i.e., the starting of the maneuver) is set as the initial condition. Panels (a)–(e) represent the time series of the state variables, while (f) shows the change of curvature in time along the entire path. The path segments are separated by brown dashed lines in each panel. Real vehicles have a physical limit to the steering angle. Let this limit be $\delta_{\max} = 40^\circ = 0.7$ rad, as also marked with gray dashed line in panel (d). Three distinct methods for tuning the control gains are presented: the green thick line represents the case when the gains are adaptively tuned to the most stable setup according to the instantaneous curvature; the blue thin line is the most stable for the largest occurring curvature value; while the orange dashed line is the most stable for the zero curvature, i.e. assuming straight-line motion.

Fig. 2(b) also helps visualize the motion in the case of adaptive gain tuning that provides a smooth and precise parking maneuver. In panel (c), the motion is illustrated when the gains of the controller are tuned for $\kappa = 0$. The motion becomes unstable as the steering angle exceeds its limitation and the truck overturns in this latter case.

As a main contribution, using adaptive control gain tuning is recommend to solve the reversing problem along the path of varying curvature. However, considering the largest occurring curvature in the gain tuning method can also lead to an acceptable solution, although precise positioning may not be achievable at the straight-line segment (when $\kappa = 0$). Finally, neglecting the path curvature in the 90-degree alley dock maneuver in reverse can easily lead to unstable motion.

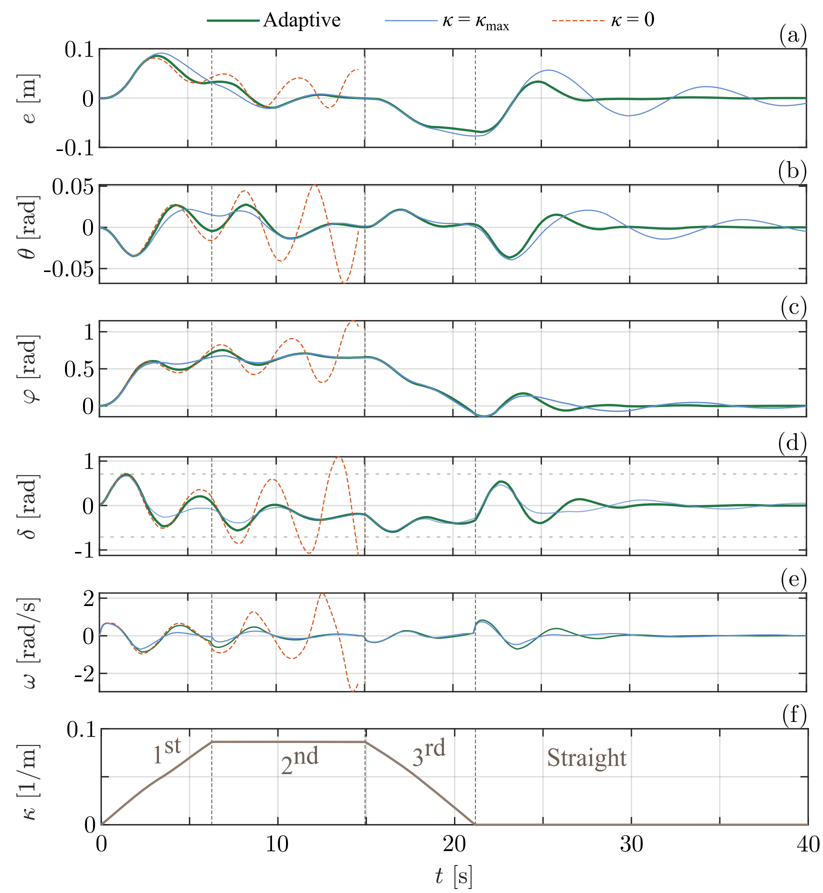

Fig. 3: Simulated time series of the state variables during the 90-degree alley dock (i.e., reversing along a path with varying curvature) with time delay $\tau = 0.5$ s in order to demonstrate the differences between the three distinct gain tuning methods. The control gain related to the lateral deviation is $P_e = -5$ rad/m. Vertical dashed lines separate the segments of the path. Horizontal dashed lines in (d) mark the physical limit of the steering angle.

4 Conclusion

A possible control algorithm for reversing a truck-semitrailer combination along a path was constructed using the single-track kinematic model. The time delay in the control loop and the dynamics of the steering mechanism were also included in the model. An adaptive control gain tuning method was established besides two other strategies by which reversing may be stabilized even in case of a large time delay. These methods were implemented via a typical parking maneuver involving varying path curvature. In order to achieve the best performance in solving the path-following problem, the adaptive control scheme is recommended based on the presented comparison.

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